Application of new multiloop QCD input to the analysis of xF_3 data

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ABSTRACT

The new theoretical input to the analysis of the experimental data of the CCFR collaboration for F_3 structure function of νN deep inelastic scattering is considered. This input comes from the next-to-next-to-leading order corrections to the anomalous dimensions of the Mellin moments of the F_3 structure function. The QCD scale $\Lambda_{\overline{MS}}^{(4)}$ is extracted from higher-twist independent fits. The results obtained demonstrate the minimization of the influence of perturbative QCD contributions to the value of $\Lambda_{\overline{MS}}^{(4)}$.

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Abstract

The new theoretical input to the analysis of the experimental data of the CCFR collaboration for F_3 structure function of νN deep inelastic scattering is considered. This input comes from the next-to-next-to-leading order corrections to the anomalous dimensions of the Mellin moments of the F_3 structure function and N^{3}LO corrections to the related coefficient funtions. The QCD scale parameter $\Lambda^{(4)}_{\overline{MS}}$ is extracted from higher-twist independent fits. The results obtained demonstrate the minimization of the influence of perturbative QCD contributions to the value of $\Lambda^{(4)}_{\overline{MS}}$.

1 Introduction

One of the most important current problems of symbolic perturbative QCD studies is the analytical evaluation of the next-to-next-to-leading order (NNLO) QCD corrections to the kernels of the DGLAP equations [1] for different structure functions of the deep-inelastic scattering (DIS) process. In this note we will apply the related information for the fixation of definite uncertainties of the NNLO analysis [2, 3] of experimental data for F_3 structure function (SF) data of νN DIS, provided by the CCFR collaboration [4] at the Fermilab Tevatron and present preliminary results of our improved fits which will be described elsewhere [5].

2 Methods of analysis of DIS data

There are several methods of analysis of the experimental data of DIS in the high orders of perturbation theory. The traditional method is based on the solution of the DGLAP equation, which in the case of the F_3 SF has the following form:

$$Q^2 \frac{d}{dQ^2} F_3(x, Q^2) = \frac{1}{2} \int_x^1 \frac{dy}{y} \left[V_{F_3}(y, A_s) + \beta(A_s) \frac{\partial \ln C_{F_3}(y, A_s)}{\partial A_s} \right] F_3\left(\frac{x}{y}, Q^2 \right) \tag{1}$$

where $A_s = \alpha_s/(4\pi)$, $\mu \partial A_s/\partial \mu = \beta(A_s)$ is the QCD β-function and $C_{F_3}(y, A_s)$ is the coefficient function, defined as

$$C_{F_3}(y, A_s) = \sum_{n \geq 0} C_{F_3,n}(y)\left(\frac{\alpha_s}{4\pi}\right)^n \tag{2}$$

and $V_{F_3}(z)$ is the DGLAP kernel, related to a non-singlet (NS) F_3 SF. The solution of Eq.(1) is describing the predicted by perturbative QCD violation of scaling [6] or automedeling [7] behaviour of the DIS SFs by the logarithmically decreasing order α_s-corrections.

The coefficient function we are interested in has been known at the NNLO for quite a long period. The term $C_{F_3,2}(y)$ was analytically calculated in Ref.[8]. The results of these calculations were confirmed recently [9] using a different technique.

The kernel $V_{F_3}(z, \alpha_s)$ is analytically known only at the NLO. However, since there exists a method of symbolic evaluation of multiloop corrections to the renormalization group functions in the $\overline{MS}$-scheme [10] and its realization at the FORM system, it became possible to calculate analytically the NNLO corrections to the $n = 2, 4, 6, 8, 10$ Mellin moments of the NS kernel of the F_2 SF [11]. They have the following expansion:

$$-\int_0^1 z^{n-1} V_{NS,F_2}(z, \alpha_s)dz = \sum_{i \geq 0} \gamma_{NS,F_2}^{(i)}(n)\left(\frac{\alpha_s}{4\pi}\right)^{i+1} \tag{3}$$

and are related to the anomalous dimension of NS renormalization group (RG) constants of F_2 SF[1] :

$$\mu \frac{\partial \ln Z_n^{NS,F_2}}{\partial \mu} = \gamma_{NS,F_2}^{(n)}(\alpha_s) \qquad . \tag{4}$$

These results were used in the process of the fits of Refs.[2, 3] of the CCFR data for the F_3 SF with the help of the Jacobi polynomial method [13]. It allows the reconstruction of the SF F_3 from the **finite** number of Mellin moments $M_{j,F_3}(Q^2)$ of the xF_3 SF:

$$F_3^{Nmax}(x, Q^2) = w \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_{j+2,F_3}^{TMC}(Q^2) \tag{5}$$

where $w = w(\alpha, \beta) = x^{\alpha-1}(1-x)^\beta$, $\Theta_n^{\alpha,\beta}$ are the orthogonal Jacobi polynomials and $c_j^{(n)}(\alpha, \beta)$ is the combination of Euler Γ-functions, which is factorially increasing with increasing of N_{max} and thus n.

The expressions for $M_{j+2,F_3}^{TMC}(Q^2)$ include the information about Mellin moments of the coefficient function

$$C_{n,F_3}(Q^2) = \int_0^1 x^{n-1} C_{F_3}(x, \alpha_s)dx = \sum_{i \geq 0} C^{(i)}(n)\left(\frac{\alpha_s}{4\pi}\right)^i \tag{6}$$

where $C^{(0)}(n) = 1$. The target mass corrections, proportional to $(M_N^2/Q^2)M_{j+4,F_3}(Q^2)$, are also included into the fits. Therefore, the number of the Jacobi polynomials $N_{max} = 6$ corresponds

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[1]The method of renormalization group was originally developed in [12].

to taking into account the information about RG evolution of 10 moments, and $N_{max} = 9$ presumes that the evolution of $n = 13$ number of Mellin moments is considered.

The procedure of reconstruction of $F_3(x, Q^2)$ from the finite number of Mellin moments and the related fits of the experimental data were implemented in the form of FORTRAN programs. The details of the fits of the CCFR data, based on RG evolution of 10 moments, are desribed in Refs.[2, 3] (for the brief review see Ref.[14]). In the process of these analyses the following approximations were made: a) it was assumed that for a large enough number of moments, $\gamma_{NS,F_3}^{(n)}(\alpha_s) \approx \gamma_{NS,F_2}^{(n)}(\alpha_s)$; b) since the odd NNLO terms of $\gamma_{NS,F_2}^{(n)}$ are explicitly unknown, they were fixed using the smooth interpolation procedure proposed in Ref.[15]. It was known that the additional contributions, proportional to the $d^{abc}d^{abc}$ structure of the colour gauge group $SU(N_c)$ are starting to contribute to the coefficients of $\gamma_{NS,F_3}^{(n)}(\alpha_s)$ from the NNLO [3]. In the process of the analysis of Refs.[2, 3] it was assumed that they were not dominating and therefore were not taken into account.

3   New inputs for the fits

After recent explicit analytical evaluation of the NNLO coefficients of $\gamma_{NS,F_3}^{(n)}(\alpha_s)$ at $n =$3,5,7 ,9,11,13 (see Ref.[16]) it became possible to fix this uncertainty (it is worth noting that the NNLO contribution to $\gamma_{NS,F_2}^{(n)}(\alpha_s)$ for $n =$12 was analytically evaluated in Ref.[16] also). To estimate the NNLO terms of $\gamma_{NS,F_3}^{(n)}(\alpha_s)$ at $n =$4,6,8,10,12 we applied the smooth interpolation procedure, identical to the one used to estimate the odd NNLO terms of $\gamma_{NS,F_2}^{(n)}(\alpha_s)$, while the numerical value of $\gamma_{NS,F_3}^{(2)}(2)$ was fixed with the help of an extrapolation procedure, where we have not used the value at $n = 1$. The justification and more details of this procedure will be given elsewhere [5].

The used numerical results of the NNLO contributions $\gamma_{NS,F_3}^{(2)}(n)$ with and without $d^{abc}d^{abc}$-factors are presented in Table 1, where we marked in parenthesis the estimated even terms. The expressions for the NNLO contributions to the NS anomalous dimensions terms $\gamma_{NS,F_2}^{(2)}(n)$ are also given for comparison. They include the numerical results of the explicit analytical calculations of Refs.[11, 16], normalized to $f = 4$ numbers of active flavours, and the results of the smooth interpolation procedure, in parenthesis, applied for estimating explicitly uncalculated odd terms. The satisfactory agreement between the numbers in the second and third columns supports the assumptions a) and b) mentioned above.

In Table 2 the numerical expressions for the coefficients of Eq.(6) for $f = 4$ numbers of active flavours are given. They include the results of explicit calculations of N³LO corrections of odd moments [16], supplemented with the information about the coefficients of the Gross–Llewellyn Smith sum rule [17, 18], defined by the $n = 1$ Mellin moment of the xF_3 SF. The numbers in parenthesis are the results of the interpolation procedure. In the last column we present the values of $C^{(3)}(n)$, obtained with the help of the [1/1] Padé estimates approach. One can see that the agreement of Padé estimates with the used N³LO results is good in the case

n	$\gamma_{NS,F_3}^{(2)}(n)$	$d^{abc}d^{abc}$ neglected in $\gamma_{NS,F_3}^{(2)}(n)$	$\gamma_{NS,F_2}^{(2)}(n)$
2	(631)	(585)	612.06
3	861.65	836.34	(838.93)
4	(1015.37)	(1001.42)	1005.82
5	1140.90	1132.73	(1135.28)
6	(1247)	(1241.21)	1242.01
7	1338.27	1334.32	(1334.65)
8	(1420)	(1416.73)	1417.45
9	1493.47	1491.13	(1492.02)
10	(1561)	(1558.85)	1559.01
11	1622.28	1620.73	(1619.83)
12	(1679.81)	(1677.70)	1678.40

Table 1: The numerical expressions of the NNLO coefficients of anomalous dimensions of the n-th NS moments of the F_3 and F_2 SFs at $f = 4$. The numbers in parenthesis are the estimated results.

n	$C^{(1)}(n)$	$C^{(2)}(n)$	$C^{(3)}(n)$	$C^{(3)}(n)_{[1/1]}$
1	-4	-52	-644.35	-676
2	-1.78	-47.47	(-1127.45)	-1268
3	1.67	-12.72	-1013.17	97
4	4.87	37.12	(-410.66)	283
5	7.75	95.41	584.94	1175
6	10.35	158.29	(1893.58)	2421
7	12.72	223.90	3450.47	3940
8	14.90	290.88	(5205.39)	5679
9	16.92	358.59	7120.99	7602
10	18.79	426.44	(9170.21)	9677
11	20.55	494.19	11332.82	11884
12	22.20	561.56	(13590.97)	14205
13	22.76	628.45	15923.91	17353

Table 2: The numerical expressions for the coefficients of the coefficient functions for n-th Mellin moments of the F_3 SF up to N³LO and their [1/1] Padé estimates.

	N_{max}	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)
result of Ref.[3]: NLO	6	339±36
	7	340±37
	8	343±37
	9	345±37
	10	339±36
NNLO	6	326±35
NNLO results with	6	325±35
inclusion of NNLO	7	326±31
terms of $\gamma^{(n)}_{NS,F_3}$	8	329±36
	9	332±36
N^{3}LO approximate results with	6	324±33
inclusion of the interpolated	7	322±33
values of $C^{(3)}$(n)-terms	8	325±34
	9	326±33

Table 3: The results of the fits of the CCFR data for xF_3 SF, taking into account the NNLO approximation for $\gamma^{(n)}_{F_3,NS}$. The initial scale of RG evolution is Q_0^2=20 GeV^2.

of the Gross–Llewellyn Smith sum rule (this fact was already known from the considerations of Ref.[19]). In the case of $n = 2$ and $n \geq 6$ moments the results are also in satisfactory agreement. Indeed, one should keep in mind that the difference between the results of column 3 and 4 of Table 2 should be devided by the factor $(1/4)^3$, which comes from our definition of expansion parameter $A_s = \alpha_s/(4\pi)$. Note, that starting from $n \geq 6$ the results of application of [0/2] Padé approximants, which in accordance with analysis of Ref.[20] are reducing scale-dependence uncertainties, are even closer to the the results of the interploation procedure (for the comparison of the estimates, given by [1/1] and [0/2] Padé approximants in the case of moments of xF_3 SF see Ref.[3], while in Ref.[21] the similar topic was analysed within the quantum mechanic model). For $n =3,4$ the interpolation method gives completely different results. The failure of the application of the Padé estimates approach in these cases might be related to the irregular sign structure of the perturbative series under consideration.

4 Some results of the fits

In Table 3 we present the comparison of the results of the determination of the $\Lambda^{(4)}_{\overline{MS}}$ parameter, made in Ref.[3], with the new ones, obtained by taking into account more definite theoretical information. Since NNLO corrections to the anomalous dimensions and N^{3}LO contributions to the coefficient functions of odd moments of the xF_3 SF are now known up to $n =13$, it became possible to study the dependence of the results of the fits from the value of N_{max}, which we can now vary from $N_{max} = 6$ to $N_{max} = 9$. It should be mentioned that for $N_{max} = 6$ the new NNLO result and its Q_0^2 dependence are in agreement with the results of Ref.[3]. However, the incorporation of higher number of moments, and thus the increase of N_{max}, make the NNLO

(and approximate N^{3}LO) results almost independent from the variation of Q_0^2 in the interval 5 GeV^2–100 GeV^2. This is the welcome feature of including into the fits the results of the new analytical calculations of the NNLO corrections to anomalous dimensions and N^{3}LO corrections to the coefficient functions of odd moments of the xF_3 SF [16]. Comparing now the central values of the results of the stable NLO fits of Ref.[3] with the new NNLO and N^{3}LO results, we observe the decrease of the theoretical uncertainties and, probably, the saturation of the predictive power of the corresponding perturbative series at the 4-loop level. More detailed results of our fits, including extraction of $\alpha_s(M_Z)$, its scale dependence and the information about the behaviour of twist-4 corrections at the NNLO and N^{3}LO, in the case of $N_{max} = 9$, will be described elsewhere [5].

Acknowledgements

References

[1] Gribov, V. N., and Lipatov L. N., *Sov. J. Nucl. Phys.* **15** (1972) 438; Lipatov L. N., *Sov. J. Nucl. Phys.* **20** (1975) 94;
Altarelli G., and Parisi. G, *Nucl. Phys.* **B126** (1977) 298;
Dokshitzer Yu. L., *Sov. Phys. JETP* **B126** (1977) 641.

[2] Kataev A. L., Kotikov A. V., Parente G., and Sidorov A. V., *Phys. Lett.* **B417** (1998) 374.

[3] Kataev A. L., Parente G., and Sidorov A. V., *Nucl. Phys.* **B 573** (2000) 405.

[4] CCFR-NuTeV Collab., Seligman W.G. et al., *Phys. Rev. Lett.* **79** (1997) 1213.

[5] Kataev A. L., Parente G., and Sidorov A. V., work in progress.

[6] Bjorken J. D., *Phys. Rev.* **179** (1969) 1547.

[7] Bogolyubov N. N., Vladimirov V. S., Tavkhelidze A. N., *Teor. Mat. Fiz.* **12** (1972) 3.

[8] Zijlstra E.B., and van Neerven W.L., *Phys. Lett.* **B297** (1992) 377.

[9] Moch S., and Vermaseren J. A. M., *Nucl. Phys.* **B573** (2000) 853.

[10] Tkachov F. V., *Phys. Lett.* **B100** (1981) 65;
Chetyrkin K. G., and Tkachov F. V., *Nucl. Phys.* **B192** (1981) 159.

[11] Larin S. A., van Ritbergen T., and Vermaseren J. A. M., *Nucl. Phys.* **B427** (1994) 41;
Larin S. A., Nogueira P., van Ritbergen T., and Vermaseren J. A. M., *Nucl.Phys.* **B492** (1997) 338.

[12] Stueckelberg E.C.G. and Petermann A. *Helv. Phys. Acta* **26** (1953) 499;
Gell-Mann M. and Low F. E., *Phys. Rev.* **95** (1954) 1300;
Bogoliubov N. N. and Shirkov D. V., *Nuov. Cim.* **3** (1956) 845.

[13] Parisi G., and Sourlas N., *Nucl. Phys.* **B151** (1979) 421;
Barker I. S., Langiensiepen C. B., and Shaw G., *Nucl. Phys.* **B186** (1981) 61;
Chýla J., and Rames J., *Z. Phys.* **C31** (1986) 151;
Krivokhizhin V. G. et al., *Z. Phys.* **C36** (1987) 51; ibid. **C48** (1990) 347.

[14] Kataev A. L., Parente G., and Sidorov A. V., Preprint DTP/00/02 [hep-ph/0001096]; Proceedings of Bogolyubov Conference "Problems of Theoretical and Mathematical Physics", October 1999; *Phys. Part. Nucl. (Fiz. Elem. Chast. Atom. Yadra)*, **31** part7b (2000) 29. Catani S. et al., *QCD* Preprint CERN-TH-2000-131 [hep-ph/0005025]; in *Proceedings of the Workshop on Standard Model (and more) at the LHC*, Eds. G. Altarelli and M.L. Mangano; CERN Yellow Report 2000-004 (2000).

[15] Parente G., Kotikov A. V., and Krivokhizhin V. G., *Phys. Lett.* **B333** (1994) 190.

[16] Retey A., and Vermaseren J. A. M., Preprint TTP00-13, NIKHEF-2000-018 [hep-ph/0007294]

[17] Gorishny S. G., and Larin S. A., *Phys.Lett.* **B172** (1986) 109.

[18] Larin S. A., and Vermaseren J. A. M., *Phys. Lett.* **B259** (1991) 345.

[19] Samuel M.A., Ellis J., and Karliner M., *Phys.Rev.Lett.* **74** (1995) 4380.

[20] Gardi E., *Phys. Rev.* **D56** (1997) 68 [hep-ph/9611453].

[21] Penin A. A. and Pivovarov A. A., *Phys. Lett.* **B367** (1996) 342 [hep-ph/9506370].